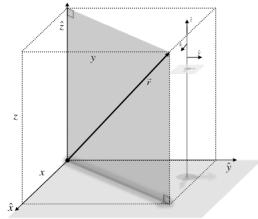


## Appendix E Curvilinear Coordinate Systems

### Cartesian Coordinates



$$\begin{aligned}\hat{x} \times \hat{y} &= \hat{z} \\ \vec{r} &= x \hat{x} + y \hat{y} + z \hat{z} \\ r &= \sqrt{x^2 + y^2 + z^2}\end{aligned}$$

$$\delta \vec{l} = \delta x \hat{x} + \delta y \hat{y} + \delta z \hat{z}$$

$$\begin{aligned}\delta \vec{A} &= \delta z \delta y \hat{x} \\ &+ \delta y \delta z \hat{y} \\ &+ \delta x \delta y \hat{z}\end{aligned}$$

$$\delta V = \delta x \delta z \delta y$$

$$\begin{aligned}\vec{\nabla} \cdot \vec{F} &= \frac{\partial}{\partial x} F_x \\ &+ \frac{\partial}{\partial y} F_y \\ &+ \frac{\partial}{\partial z} F_z\end{aligned}$$

$$\vec{\nabla} f = \frac{\partial f}{\partial x} \hat{x} + \frac{\partial f}{\partial y} \hat{y} + \frac{\partial f}{\partial z} \hat{z}$$

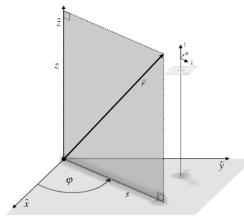
$$\begin{aligned}\vec{\nabla} \times \vec{F} &= \left( \frac{\partial}{\partial y} F_z - \frac{\partial}{\partial z} F_y \right) \hat{x} \\ &+ \left( \frac{\partial}{\partial z} F_x - \frac{\partial}{\partial x} F_z \right) \hat{y} \\ &+ \left( \frac{\partial}{\partial x} F_y - \frac{\partial}{\partial y} F_x \right) \hat{z}\end{aligned}$$

$$\begin{aligned}x &= s \cos \varphi = r \sin \theta \cos \varphi \\ y &= s \sin \varphi = r \sin \theta \sin \varphi \\ z &= z = r \cos \theta\end{aligned}$$

$$\begin{aligned}\hat{x} &= \cos \varphi \hat{s} - \sin \varphi \hat{\phi} \\ \hat{y} &= \sin \varphi \hat{s} + \cos \varphi \hat{\phi} \\ \hat{z} &= \hat{z}\end{aligned}$$

$$\begin{aligned}\hat{x} &= \sin \theta \cos \varphi \hat{r} + \cos \theta \cos \varphi \hat{\theta} - \sin \varphi \hat{\phi} \\ \hat{y} &= \sin \theta \sin \varphi \hat{r} + \cos \theta \sin \varphi \hat{\theta} + \cos \varphi \hat{\phi} \\ \hat{z} &= \cos \theta \hat{r} - \sin \theta \hat{\theta}\end{aligned}$$

### Cylindrical Coordinates



$$\begin{aligned}\hat{s} \times \hat{\phi} &= \hat{z} \\ \vec{r} &= s \hat{s} + z \hat{z} \\ r &= \sqrt{s^2 + z^2}\end{aligned}$$

$$\delta \vec{l} = \delta s \hat{s} + s \delta \varphi \hat{\phi} + \delta z \hat{z}$$

$$\begin{aligned}\delta \vec{A} &= s \delta \varphi \delta z \hat{s} \\ &+ \delta z \delta s \hat{\phi} \\ &+ s \delta \varphi \delta s \hat{z}\end{aligned}$$

$$\delta V = s \delta s \delta \varphi \delta z$$

$$\begin{aligned}\vec{\nabla} \cdot \vec{F} &= \frac{1}{s} \frac{\partial}{\partial s} (s F_s) \\ &+ \frac{1}{s} \frac{\partial}{\partial \varphi} F_\varphi \\ &+ \frac{\partial}{\partial z} F_z\end{aligned}$$

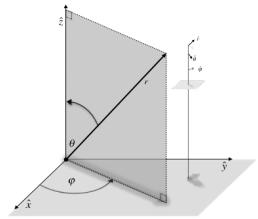
$$\vec{\nabla} f = \frac{\partial f}{\partial s} \hat{s} + \frac{1}{s} \frac{\partial f}{\partial \varphi} \hat{\phi} + \frac{\partial f}{\partial z} \hat{z}$$

$$\begin{aligned}\vec{\nabla} \times \vec{F} &= \left( \frac{1}{s} \frac{\partial}{\partial \varphi} F_z - \frac{\partial}{\partial z} F_\varphi \right) \hat{s} \\ &+ \left( \frac{\partial}{\partial z} F_s - \frac{\partial}{\partial s} F_z \right) \hat{\phi} \\ &+ \left( \frac{1}{s} \frac{\partial}{\partial s} (s F_\varphi) - \frac{1}{s} \frac{\partial}{\partial \varphi} F_s \right) \hat{z}\end{aligned}$$

$$\begin{aligned}s &= r \sin \theta = \sqrt{x^2 + y^2} \\ \varphi &= \arctan \left( \frac{y}{x} \right) \\ z &= r \cos \theta\end{aligned}$$

$$\begin{aligned}\hat{s} &= \cos \varphi \hat{x} + \sin \varphi \hat{y} \\ \hat{\phi} &= -\sin \varphi \hat{x} + \cos \varphi \hat{y} \\ \hat{z} &= \hat{z}\end{aligned}$$

### Spherical Coordinates



$$\begin{aligned}\hat{r} \times \hat{\theta} &= \hat{\phi} \\ \vec{r} &= r \hat{r} \\ \delta \vec{l} &= \delta r \hat{r} + r \delta \theta \hat{\theta} \\ &+ r \sin \theta \delta \theta \hat{\phi}\end{aligned}$$

$$\begin{aligned}\delta \vec{A} &= r^2 \sin \theta \delta \theta \delta \varphi \hat{r} \\ &+ r \sin \theta \delta r \delta \varphi \hat{\theta} \\ &+ r \delta \theta \delta r \hat{\phi}\end{aligned}$$

$$\delta V = r^2 \sin \theta \delta r \delta \theta \delta \varphi$$

$$\begin{aligned}\vec{\nabla} \cdot \vec{F} &= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 F_r) \\ &+ \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin(\theta) F_\theta) \\ &+ \frac{1}{r \sin \theta} \frac{\partial}{\partial \varphi} F_\varphi\end{aligned}$$

$$\vec{\nabla} f = \frac{\partial f}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \varphi} \hat{\phi}$$

$$\begin{aligned}\vec{\nabla} \times \vec{F} &= \frac{1}{r \sin \theta} \left( \frac{\partial}{\partial \theta} (\sin \theta F_\varphi) - \frac{\partial}{\partial \varphi} (r F_\theta) \right) \hat{r} \\ &+ \frac{1}{r} \left( \frac{1}{\sin \theta} \frac{\partial}{\partial \varphi} F_r - \frac{\partial}{\partial r} (r F_\varphi) \right) \hat{\theta} \\ &+ \frac{1}{r} \left( \frac{\partial}{\partial r} (r F_\theta) - \frac{\partial}{\partial \theta} F_r \right) \hat{\phi}\end{aligned}$$

$$\begin{aligned}r &= \sqrt{x^2 + y^2 + z^2} = \sqrt{s^2 + z^2} \\ \theta &= \arctan \left( \sqrt{\frac{x^2 + y^2}{z^2}} \right) = \arctan \left( \frac{s}{z} \right) \\ \varphi &= \arctan \left( \frac{y}{x} \right) = \varphi\end{aligned}$$

$$\begin{aligned}\hat{r} &= \sin \theta \cos \varphi \hat{x} + \sin \theta \sin \varphi \hat{y} + \cos \theta \hat{z} \\ \hat{\theta} &= \cos \theta \cos \varphi \hat{x} + \cos \theta \sin \varphi \hat{y} - \sin \theta \hat{z} \\ \hat{\phi} &= -\sin \varphi \hat{x} + \cos \varphi \hat{y}\end{aligned}$$